Fermilab

Is Electron Cooling Relevant to VLHC?

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Abstract

The appropriate target for the transverse emittance at injection into the VLHC is discussed along with the expectations for emittance growth throughout the injector chain. After an elementary introduction to the electron cooling technique simple cooling rate formulas are introduced which are adequate to assess the feasibility of using electron cooling to contribute to VLHC performance. Finally, the usefulness of cooling at low, medium, or high energy is considered.

1 Target Emittance and Emittance Evolution

The normalized transverse emittance including 95% of the beam is given as $15\,\pi \cdot 10^{-6}$ m in the 1998 green book[1]. This is less than the present typical value $\varepsilon_n \approx 20\,\pi$ for the Tevatron, but for a machine to be built several years from now it is an unnecessarily conservative choice. The intended value at the SSC was $6\,\pi$. To obtain this goal the design included a very difficult low energy booster design and an extremely tight budget for emittance growth along the injector chain, namely about a factor of two from the end of the linac. Nonetheless, experiments done at the Fermilab Booster[2] showed that it produced beam of the requisite brightness when collimated in the 8 GeV line from its normal $12\,\pi$ to the SSC specification.

The emittance at the end of the linac is usually not relevant; generally the energy of injection into the first circular machine and the number of turns injected set the lower limit for the emittance of the injector chain. The final emittance from the Fermilab Booster at intensity of below $2 \cdot 10^{10}$ p/bunch is about $7\,\pi$, not much worse than the linac value. However, as intensity is raised, the emittance increases linearly. The formula following gives the space charge tune shift, which is also about the tune spread:

$$\Delta\nu_{\mathrm{sc}} = -\frac{3r_{\mathrm{p}}N_{\mathrm{tot}}}{2\varepsilon_n\beta\gamma^2B} \ ,$$

where r_p is the proton classical radius, N_{tot} is the number of protons in the ring, and $B < 1$ is the bunching factor. The Booster emittance appears to grow at injection to keep $\Delta\nu_{\mathrm{sc}} \sim 0.3$. A second generation Booster could have a higher injection energy and possibly a significantly larger bunching factor. Furthermore, the constant is not necessarily the same from machine to machine. The BNL booster is reported to allow a tune shift of 0.8 when the field is carefully corrected.

The filling for the low field VLHC is $2 \times 1.27 \cdot 10^{10} \times 10^5$ protons, and a factor about 1/6 of that for a high field version. For Main Injector (MI) intensity of $3 \cdot 10^{13}$ protons/pulse, about 100 MI cycles would be required for a fill, and the intensity of the bunches in the Booster would be about $6 \cdot 10^{10}$. In this case it is clear that some charge rearrangement will be required. If one wishes to populate 53 MHz buckets with the correct charge starting at the Booster, the Booster intensity will be $\sim 10^{12}$/batch and the transverse emittance should be small, quite likely less than the $7\,\pi$ that it would be today. However, it would then take 400 MI cycles to load the collider. This is only sixteen minutes, so if the initial emittance is satisfactory, the low intensity filling is a straightforward procedure.

After the first circular machine it is probably *possible* to keep dilution through the entire chain to less than $2\times$ — not *easy*. However, if the emittance out of the Booster is something like $6\,\pi$ or can reasonably be collimated to that level, it may be possible to cool the beam to substantially lower emittance in a reasonable time. Presumably this is a consideration only for the low field case because the high field design has a larger aperture and natural radiative cooling. For the low field design, the smaller beam at injection could be advantageous for some further aperture reduction, some margin for injection error and closed orbit, and smaller number of particles required for a given luminosity. A factor of four in emittance would reduce the stored

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energy and the number of cycles required to fill by the same factor. If the necessary cooling can be accomplished in less than an hour, it becomes interesting. If done in the MI, this means about a minute per cycle; if it could be done at 3 TeV in the low-field High Energy Booster, only two or three fills would need cooling and ten to twenty minutes of cooling time might be satisfactory.

The general burden of the foregoing considerations is that it is realistic to plan for lower than $15\,\pi\mu$m normalized transverse emittance at injection into the VLHC, even in the absence of cooling. Given that lower emittance early in the injector chain it may be possible and advantageous to reduce it further by electron cooling. Some consideration will be given to specifics of electron cooling, but before a lot of detailed thinking goes into system parameters it would be useful to see what design leverage could be obtained with lower transverse emittance. For the sake of discussion, it is reasonable to examine what would be the payoff for reducing the emittance to $4\,\pi\mu$m (normalized, 95%).

2 Electron Cooling Basics

Electron beam with the lowest practicable velocity spread is passed through a straight section of an ion storage ring at the same mean velocity as the ions. In the coordinate frame moving at beam velocity, the ions move randomly among electrons of much lower energy. The multiple Coulomb scattering of each ion passes energy of random motion from the ions to the electrons. Ions may circulate for minutes or even hours through this straight section, but heated electrons are continually replaced with cooler ones. At low energies electrons are usually replaced after a single pass. At high energy electrons can be stored and cooled radiatively.

In the co-moving frame the process looks like exchange of heat between a hot ion gas and an electron gas continuously circulated at low temperature. The lab-frame energy of the electron beam is lower than the ion energy by the electron-ion mass ratio. The cooling rate is proportional to the electron current and the length of the cooling straight section.

In low energy systems cooling times of a few ms are possible. Because the Lorentz transformations that convert the cooling rate in beam-frame parameters to the laboratory frame introduce an inverse square dependence on ion energy, higher energy systems may require long cooling sections and

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high electron current.

2.1 Intuitive Model

If one goes to the beam frame to consider the non-relativistic multiple Coulomb scattering of protons by (practically) stationary unbound electrons, one can find a formula for the slowing or friction force in an elementary nuclear physics text (simplified Bethe-Bloch equation):

$$\frac{\mathrm{d}E}{\mathrm{d}z} = \frac{-4\pi n (r_e m c^2)^2}{m v_p^2} \Lambda_c = \langle F_\parallel \rangle \ ,$$

where r_e is the electron classical radius, n is the volume number density of the electrons, m is electron mass, v_p is the proton speed, and Λ_c is the so-called Coulomb logarithm, which is ~ 10 for a broad range of n and v_p. Force is $\mathrm{d}p/\mathrm{d}t$, so the average fractional rate of change of $p_\parallel$ is $\alpha_\parallel = \langle F_\parallel \rangle / p_\parallel$:

$$\alpha_\parallel \approx F_\circ \Lambda_c / (M v_p^3) \quad \text{(beam frame)},$$

where $F_\circ$ includes the charge density and fundamental constants and M is the proton mass.

Transform to the lab frame; the result is division by γ^2, one power from the time dilation in the rate and one power because the beam frame density is defined as the charge within a volume with a length contraction along beam direction. To get a formula valid for a storage ring of mean radius R and cooling section length ℓ_c it is necessary to include also a filling factor $\eta = \ell_c / 2\pi R$

$$\alpha \approx \eta F_\circ \Lambda_c / (\gamma^2 M v_p^3) \quad \text{(lab frame)}.$$

This formula is actually useful for estimates of both longitudinal and transverse rates because it depends only on the magnitude of $\vec{v_p}$. In the case where the proton velocity is significantly greater than the electron velocity, it is better than just an order of magnitude result. For relativistic beams it is almost always the case that the distribution of longitudinal velocities is much narrower than the transverse velocity distribution. Therefore, it is not so gross to replace v_p by $v_\perp$ and get a practically equivalent alternative expression for α which is especially handy for present purposes:

$$\alpha = \frac{12\pi^3 r_p r_e \Lambda_c \eta I_e r_b^3}{\gamma^2 a^2 \beta e \varepsilon_\perp^3} \ ,$$

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where new symbols are electron beam current I_e, electron beam radius a, and proton beam radius r_b. The new form comes from putting in the fundamental constants explicitly, evaluating n in terms of I_e, and equating $v_\perp$ to $\varepsilon_\perp/r_b$. The last substitution may look strange, but the β and γ factors are correct.

However, one can not neglect electron motion for low-emittance p beam. To calculate the longitudinal cooling formula analogous to the preceding result when the electron velocities are not negligible, one must calculate the force averaged over the relative velocities and take the component in the beam direction, the z-direction:

$$
\begin{aligned}
\frac{\mathrm{d}E}{\mathrm{d}z} &= F_\circ \Lambda_c \int g(\vec{v_e}) \frac{\vec{u}}{u^3} \cdot \hat{z} \, \mathrm{d}^3 \vec{v_e} \\
&= F_\circ \Lambda_c \int g(\vec{v_e}) \frac{u_\parallel}{u^3} \, \mathrm{d}^3 \vec{v_e} \ ,
\end{aligned}
$$

where $\vec{u}$ is the relative velocity. Then,

$$
\alpha_\parallel = F_\circ \Lambda_c / M \int g(\vec{v_e}) u^{-3} \, \mathrm{d}^3 \vec{v_e} = F_\circ \Lambda_c / (M \langle u^3 \rangle) \quad \text{(beam\ \ frame)}.
$$

If averaging $\langle \cdots \rangle$ is carried out also over betatron phase and, for bunched beam, synchrotron phase, the above result is correct for the case in which there is little or no longitudinal magnetic field in cooling region and is a good approximation in any case, regardless of the ratio between the proton and electron velocities. The formula for the transverse cooling rate is obtained from a similar integral for the average transverse force. The averages can be taken analytically for transverse velocities $\gg$ longitudinal, appropriate for relativistic beams, with some small approximation resulting from joining limiting results by simple smooth functions. As it turns out[3], the simple approximation given first remains appropriate for the transverse cooling in velocity ranges of interest. The longitudinal result is more affected by the refinements but is not of immediate interest for present arguments.

3   Electron Cooling at Various Beam Energies

As mentioned earlier, the physics in the beam frame is somewhat independent of the beam energy, but the isotropy of the beam velocity distributions, which, after all, originate more or less at rest in the lab frame, and the technical means to the required electron beam energy are strongly affected.

Also the the Lorentz transformations to the lab frame result not only in a γ^{-2} penalty in rate but also in a γ^{-1} tightening of angular alignment tolerance. These factors make the prospect of cooling at high energy speculative. At what may be called medium energy the challenges are also considerable, but a first project is in development[4], and some others are being discussed seriously. The usefulness of cooling for the VLHC at different energy steps is discussed briefly below.

3.1 Low Energy Cooling

Low energy will be defined as $\gamma < 2$ somewhat arbitrarily, but the upper limit is related to the fact that the technical approach used in existing coolers becomes practically impossible somewhere around this value. This is the regime in which the technique can be considered well established and where careful observations have generally validated the physics of the process in the beam frame. Nearly all experience and experts come from work in this energy region.

In Fig. 1 there is a drawing of a classical electron cooler, this one installed in LEAR. The dominant features are an electron gun inside a solenoid with a Cockcroft-Walton type high voltage supply, a toroid for bending the beam onto the cooling section, a solenoid surrounding the cooling section, an exit toroid, and a final solenoid with a decelerating collector for energy recovery. Typical applications look for very short cooling times and very low resulting emittance. So-called magnetized cooling (with $B_\parallel \sim 1$ kG) is often important.

The energy range is too low to correct emittance dilution from most of the injector chain so probably it is not helpful for VLHC. It does not get beyond the energy of important space charge effects. However, some of the experience is helpful in developing cooling systems for higher energies, and, despite technical differences, coolers for somewhat higher energies do have some general resemblance.

3.2 Medium Energy Cooling

As for the low energy case, the arbitrary upper limit for the medium energy range is set by technical considerations. It becomes possible to make a cooler using an electron storage ring with enhanced radiative cooling somewhere about $\gamma = 70$ and above. The medium energy region so defined between
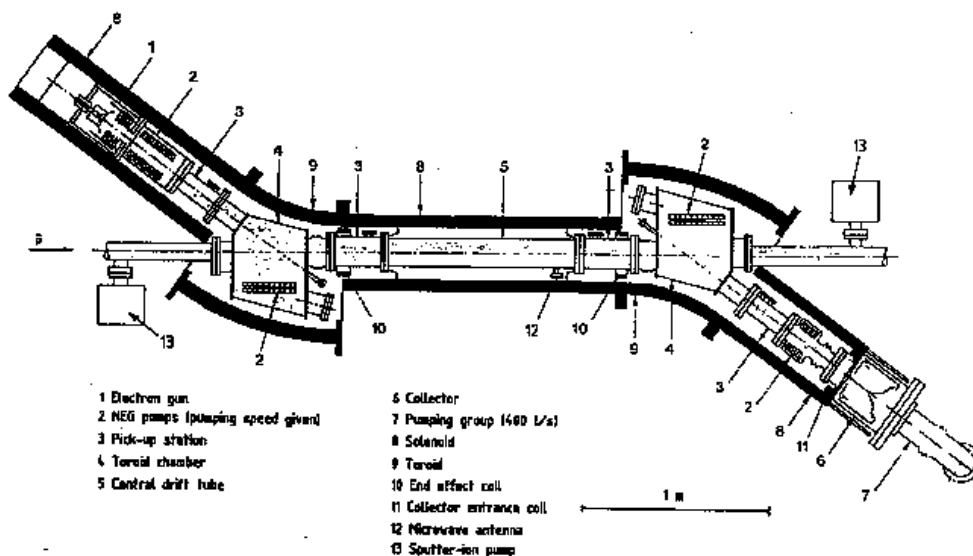
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Figure 1: A more or less typical low energy electron cooler (from the CERN LEAR antiproton storage ring)

$\gamma = 2$ and $\gamma = 70$ includes the range from injection into the MI to something beyond MI transition. At some point in this range it should be possible to cool in an acceptable time and provide an emittance which persevere to VLHC injection with small dilution.

For $\gamma \sim 10$ a dc system like the one being developed for the Recycler could be used on coasting beam. Subsequent bunching could be practically adiabatic. The Recycler cooler is not sufficient to cool $3 \cdot 10^{15}$ transversely in time short compared to store time. However, VLHC is not tomorrow, and both injector and cooling technology and experience should continue to develop. The high voltage source for the Recycler system is a 5 MV Pelletron installed outside of the MI tunnel; it is sketched in Fig. 2. The beam line and cooling section are given schematically in Fig. 3. Notice that the longitudinal magnetic field is not continuous in this design. This difference has strong implications for the beam optics, but the associated problems are ameliorated somewhat by the fact that, because of the reduction of the space charge force with energy and the irrelevance of magnetized cooling, the solenoid is less

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than 100 G.

For present Booster emittance, 140 A · m of electron beam in a straight section with $\beta_x = \beta_y = 100$ m give 90 s cooling time. Such a system could be introduced into the Main Injector. Remember that there are big rate gains to be made from modest improvement in initial emittance; the rate goes as $\varepsilon_{\text{init}}^{-3}$. As emittance goes down the beam required for a given luminosity drops, so one saves injection time in that way, and, of course, there is the reduction in stored energy of the beams, which may be a very useful side effect.

Note assumption that charge rearrangement will permit filling the VLHC with $< 2R_{\text{VLHC}}/R_{\text{MI}}$ MI loads. At $3 \cdot 10^{13}$ per MI load and $10^5 \times 2 \times 1.27 \cdot 10^{10}$ for the VLHC, about 100 MI cooling cycles are required. It would be nice (but hard) to get cooling time below 1 min. Time required depends very strongly on initial emittance but very little on final emittance desired so long as it is somewhat above the assymptotic value. Once one has a system that works at all, one can choose final emittance over a reasonable range.

At the upper end of the medium energy range bunched protons would probably be cooled by bunched electrons from a linac. It is also likely that the electrons would be recycled over several turns using an electron storage ring to lower the linac duty factor. in Fig. 4 is a schematic of this approach developed for cooling $15 - 20$ GeV protons in PETRA. Fig . 5 shows how bunches spaced at the proton spacing are accelerated in a high frequency electron linac and then rotated with a subharmonic debuncher to span the proton bunch length and simultaneously reduce the electron momentum spread. The cooler suggested for 20 GeV protons in PETRA gives 10 min cooling time; the system performance would need to be enhanced by $\sim 10\times$ to cool protons above transition in the Main Injector for VLHC.

3.3 High Energy Cooling

The damping time for the low field option is around 80 hours. To counteract intrabeam scattering and other diffusion, it appears that about 3 hour cooling time is needed. In figure six there appears an old cartoon of a concept for cooling at final energy.[5] According to NLC types[6], there should be no problem in cooling electrons in a damping ring, and by a phase space transformation due to Ya. Derbenev[7] one can get in the cooling section

$$\varepsilon_h^* = \varepsilon_v^* = \sqrt{\varepsilon_h \varepsilon_v} \sim 10\varepsilon_v = \varepsilon_h/10 \ \ .$$
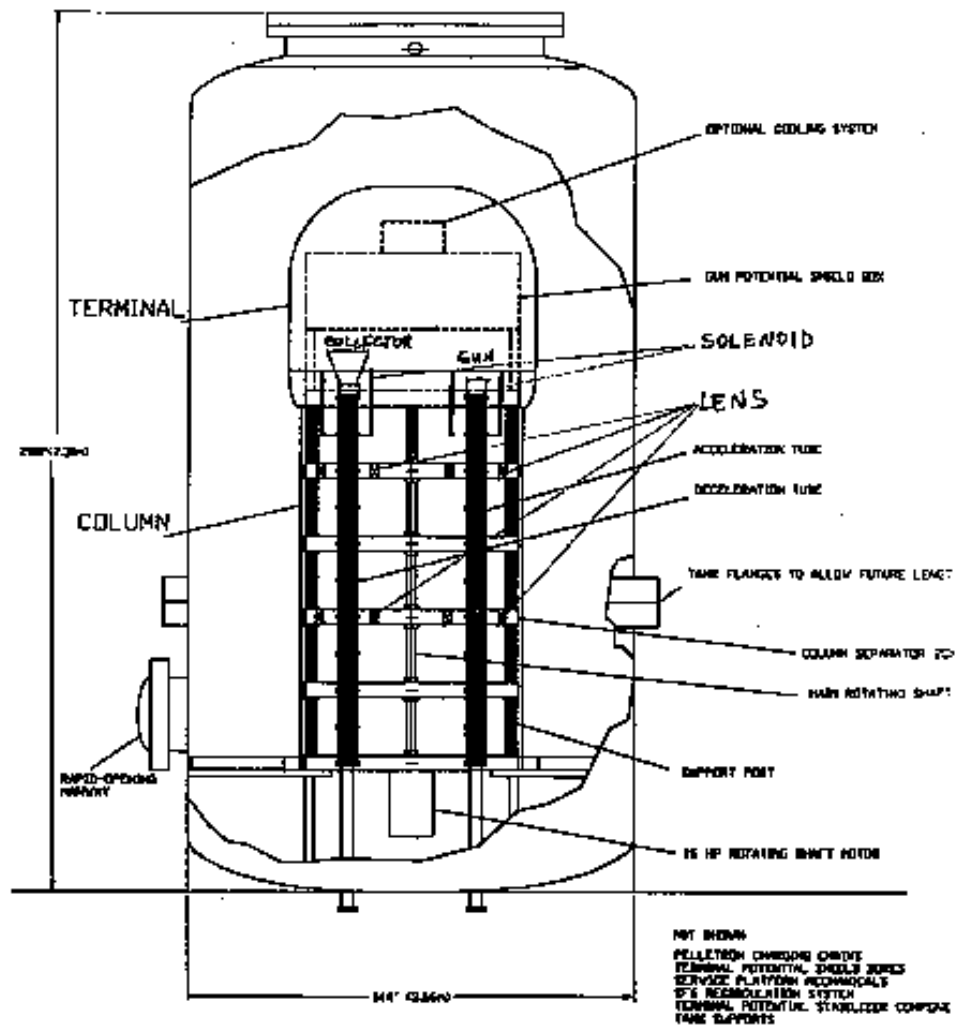
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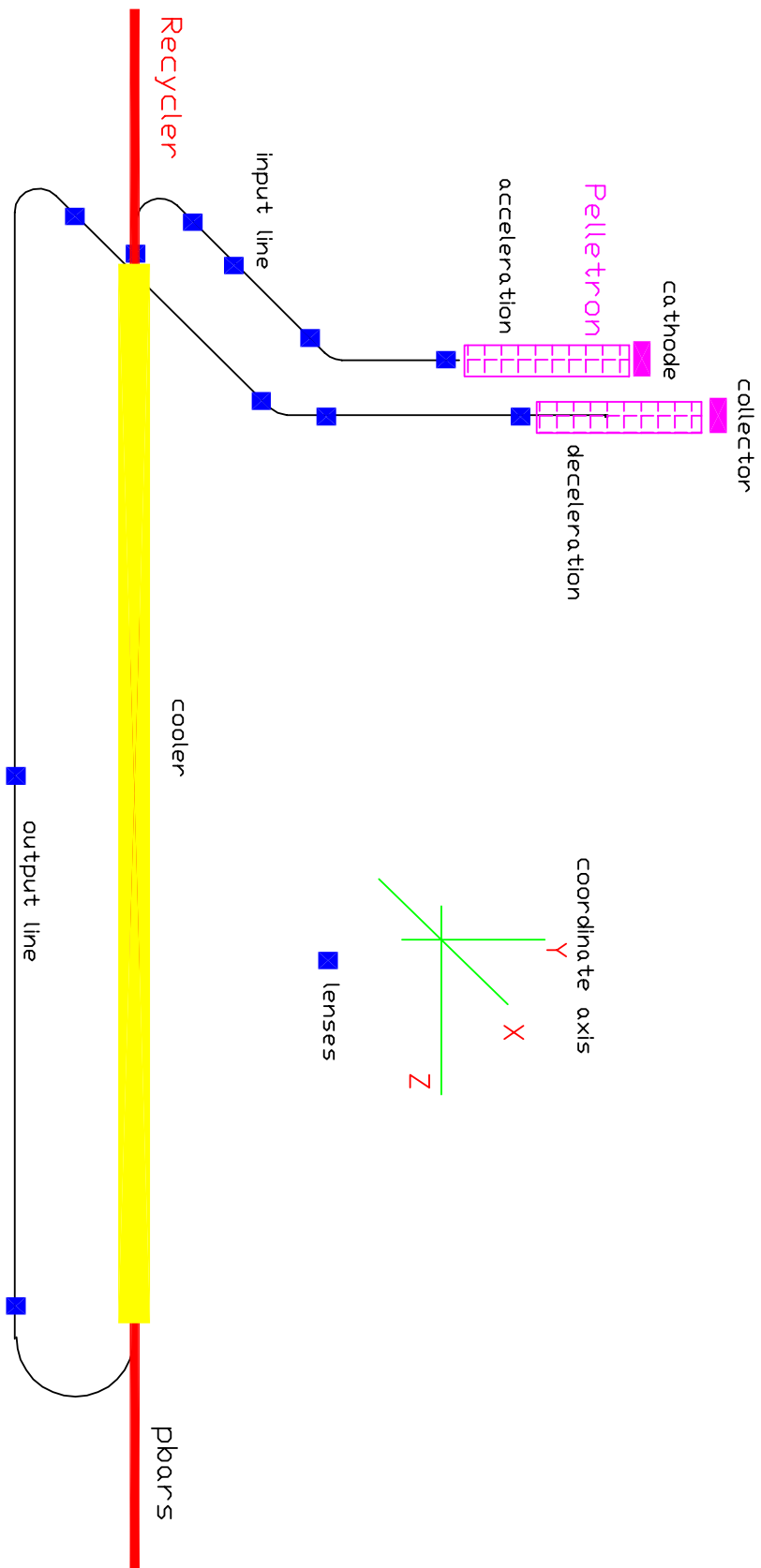Figure 2: Schematic of a 5 MV electrostatic accelerator of the Pelletron type configured for recirculating electron beam

Figure 3: The electron beamline for the Fermilab Recycler electron cooling system for accumulating 8 GeV p̄'s
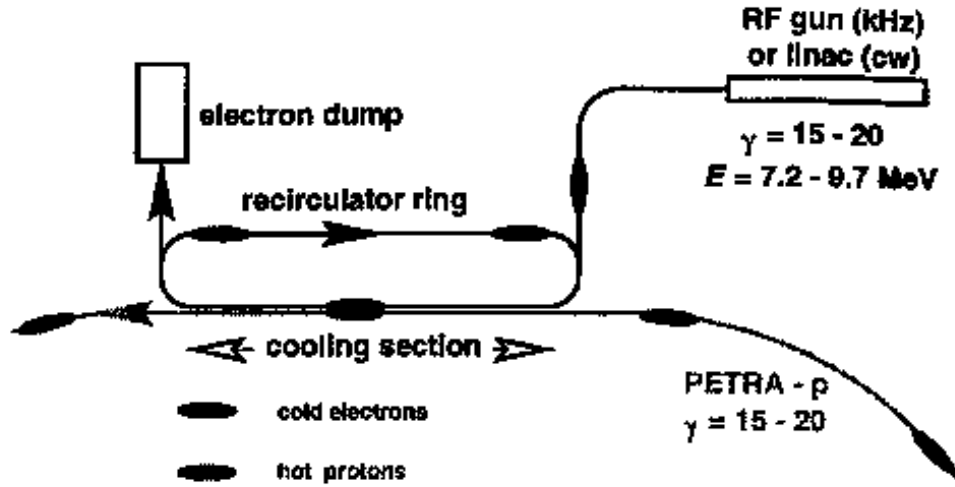
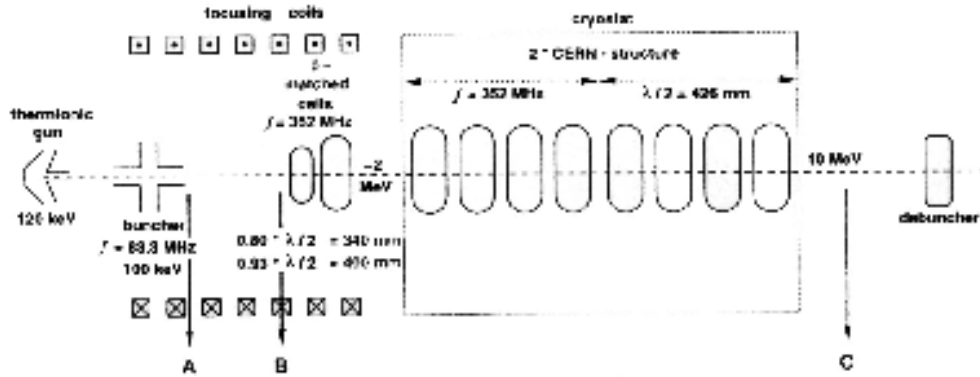Figure 4: The scheme proposed for cooling protons at 15 to 20 GeV in PETRA

Figure 5: Basic design of the linac and debunching scheme for the PETRA electron cooler
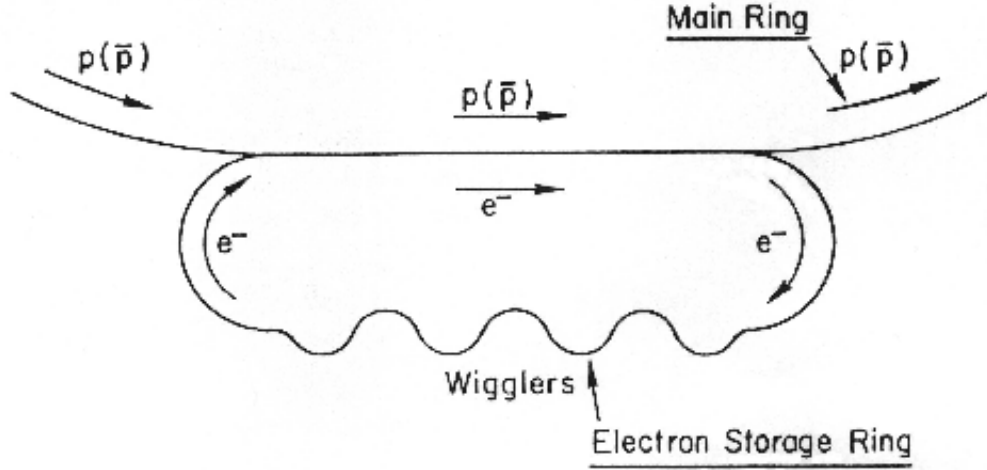
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Figure 6: A 1970's cartoon of the most promising approach to electron cooling at TeV energies (borrowed from Cline *et al.*[5])

Conceivably one could get the current and long cooling section to achieve desired cooling time, but the alignment required between the electron beam and the proton beam over a distance > 100 m is mind boggling. To retain the full cooling, the angular misalignment δ is limited

$$\delta < v_{\text{th}}/(\beta\gamma c) \ ,$$

where the thermal velocity v_{th} could be $\sim 10^{-4}c$. This seems to eliminate the possibility of cooling at 50 TeV outright. Perhaps there is some beam property like coherent transverse Schottky signals (or something) which could be used to fine-tune alignment. However, alignment at substantially better than the microradian level is not a possibility one would take for granted.

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